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The Titius-Bode Rule Revisited — Howard L. Cohen

The Titius-Bode rule approximates spacings of planetary orbits. Other "rules" can also give approximate separations for satellites orbiting some Jovian planets. Regardless of planetary or satellite spacings, it often possible to find "simple numerical rules" resembling real distances

At the April meeting of the AAC I presented a talk titled, "Should the Lord Architect Have Left That Space Empty?" Club members who attended heard a discussion of how J. Daniel Titius, an eighteenth century German astronomer, devised a numerical recipe for the orbit spacings of all planets known before the discovery of Uranus. Titius also reflected about the rule's prediction of a yet unknown planet orbiting between Mars and Jupiter.

The rule was subsequently plagiarized by Johann E. Bode, a young German astronomer, who, in his own book on astronomy, echoed the words of Titius. Bode also drew further attention to the rule (and to himself) when he used Kepler's harmonic law to predict the orbit period of the unknown planet. ("Planet X" would later turn out to be the asteroid belt.) Subsequently Bode used the discovery of Uranus in 1781 by William Herschel to illustrate the additional predictive powers of the rule.

The discovery of asteroids, beginning in 1801, which showed the average distance of the asteroid belt from the Sun fits the Titius sequence, helped guarantee the rule a place in history. This rule, which had become irrevocably linked to Bode, is now often called, "Bode's Law," although it was neither discovered by Bode nor is it a "law of nature."

The Titius-Bode rule (as I will continue to call it), correctly predicts the approximate mean distances from the Sun of all planets except Neptune. (Some authors state the rule breaks down for Neptune and Pluto but the rule, in fact, does work for Pluto *if one skips Neptune*.) To use the rule, write the sequence of numbers 0, 3, 6, 12, etc., computing each subsequent number after the three by doubling. Add four to each number and divide each result by ten to get 0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6 and 38.8. (The division by ten is simply to put the results in astronomical units or AU's — the Earth's mean distance from the Sun.) Look up the mean distances of the planets in any modern astronomy reference. Then note how each planet starting with Mercury, *including the asteroid belt at 2.8 AU*, matches the final sequence. (*Remember to skip Neptune.*)

For those who like concise mathematical notation, the Titius-Bode rule has the form $d = 0.3 \times 2^n + 0.4$ where d = distance (in AU) and $n = -\infty, 0, 1, 2, 3$, etc. One should note that the essence of the rule is doubling — as orbits increase in size, each orbit is approximately twice as large as the previous orbit (except Neptune). Some think the rule is more difficult to remember and apply than it is worth! For those who simply want to "memorize the distances of the planets," I offer "Cohen's rule" — the distances of the planets (and the asteroid belt) from the Sun, if rounded, follow a simple sequence, roughly, 1/2, 3/4, 1, 1½, 2½, 5, 10, 20, 30 and 40 AU. And this sequence *does include Neptune*!

The Titius-Bode rule is not derivable from any accepted law of nature. It is not the result of any known planetary interaction, nor is it very accurate. It is simply a “rule” for approximating distances. Originally astronomers wondered if the planet spacings showed an orderly arrangement that might point to an underlying structure to the solar system. Although few astronomers today take the rule seriously, the appeal of “regularity” remains and continues to stimulate interest in the question, “Do planet spacings say something about the origin of the solar system?”

Although astronomers are now beginning to discover other stars that apparently have planets too, we still have little knowledge of planetary spacings to judge if planet distances follow orderly “rules.” However, within our own solar system the large, gas giant planets have orbiting satellites resembling miniature solar systems. At the end of April’s talk, I asked, “Do satellite systems follow a “Titius-type” rule?” The answer is not immediately obvious since many small planetary satellites are likely captured bodies that did not form along with the parent planet in the early solar system. Hence, deciding which satellites to include in a spacing rule is difficult. Even large satellites may not have formed directly along with the parent planet or may have had their orbits drastically changed during early solar system evolution.

Jupiter is the most obvious planet to analyze since its four large Galilean satellites stand apart from Jupiter’s other much smaller orbiting bodies. Uranus also has four or possibly five satellites that stand out from its other smaller satellites. However, both Saturn and Neptune each have only one large satellite making it difficult to look for an orbit spacing rule. Finally, the smaller planets have few or no satellites making it impossible to derive a number sequence for orbit sizes. Therefore, only the Jovian and Uranian systems provide useful data to derive orbit spacing rules.

First consider the Jovian system. The following table lists distances of the Galilean satellites from Jupiter (in thousands of kilometers) and rounded distances relative to Io’s distance. The last column gives the results of applying a numerical “rule” as described below.

Satellite	**Dist. (1,000 km)**	**Dist. (rel.)**	**Rule (below)**
Io	422	1.0	1.0
Europa	671	1.6	1.5
Ganymede	1,070	2.5	2.5
Callisto	1,885	4.5	4.5

Does this system follow the Titius-Bode rule? It does not. Does it follow another recipe? Definitely yes! Although I have not seen one published, the following rule works almost perfectly: $d = 2^{n-1} + 0.5$, where $n = 0, 1, 2, 3$. That is, write the number sequence 1, 2, 4, 8, where each number is twice the previous. Add one to each and divide the result by two. The result gives the mean distances of each satellite from Jupiter relative to the innermost Galilean satellite Io. (See the last column of the table.) This rule is not only a true geometric sequence but is also simpler than the Titius rule and works for all Galilean satellites! (Note: A geometric sequence is formed by multiplying each number by a common factor to obtain the next number.)

Want to make the rule work for Amalthea, the next Jovian satellite discovered? Much smaller than the Galilean satellites (diameters from 3,140 to 4,800 km), Amalthea (diameter 170 km) remained unknown until 1892, when discovered by E. Barnard. Amalthea orbits 180 km from Jupiter or 0.4 of Io's distance. Use $n = -\infty$ (minus infinity) in the rule given above and you will get $d = 0.5$, a close match. This is equivalent to starting the number sequence with zero rather than one (viz., 0, 1, 2, 4, 8) before adding one and dividing by two.

The Uranian system is more problematic. Four or five large satellites orbit Uranus depending on whether one includes Miranda, which is about one-third to one-half the diameter of the largest four. The next table lists distances (in thousands of kilometers) of the five largest satellites from Uranus including Miranda. This table also gives the rounded distances compared with Miranda's distance. The last column again gives the results of applying a numerical "rule" as described below.

Satellite	**Dist. (1,000 km)**	**Dist. (rel.)**	**Rule (below)**
Miranda	130	1.0	1.0
Ariel	191	1.5	1.5
Umbriel	266	2.0	2.0
Titania	436	3.4	3.5
Oberon	583	4.5	4.5

A rule that "works" well is simply $d = (n + 1)/2$ where $n = 1, 2, 3$, etc. Here distances are relative to Miranda ($n = 1$). This is equivalent to writing the number sequence 1, 2, 3, etc., then adding one to each number, and finally dividing each result by two. *However, to make the rule work requires that $n = 4$, 5 and 7 apply to three yet "unknown" satellites*. Hence, use $n = 1$, 2, 3, 6, and 8 for Miranda, Ariel, Umbriel, Titania and Oberon respectively. Readers are encouraged to seek other "rules." (Let me know what you find.)

What does this all prove? Probably very little! Regardless of planetary or satellite distances, it is likely that one can find "simple numerical rules" to approximate the real spacings of orbiting celestial bodies. ☼

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